Chapter 1

Physical Quantities, Units, and Math Methods

AICE Standards: 1.1a, 1.2a, 1.2b, 1.2d

Units

- In physics, a **unit** is a standard of measurement of physical quantities with a clear definition.
- Units were invented with the intent to introduce reproducibility of experimental results.

Units

- Physical quantities are categorized by their unit.
    - Example: 10 C and 10 A are very different.
- A physical quantity *usually* consists of a numerical **magnitude** (value) and a **unit**.
    - Some quantities are actually unitless such as: coefficient of friction and index of refraction.
- Quantities without units are considered dimensionless or unitless.

SI Units

- The core 7 units in science are referred to as the SI units.
  - Meter (m) - Measures distance.
  - Second (s) - Measures time.
  - Kilogram (kg) - Measures mass.
  - Mol (mol) - Measures amount of substance.
  - Ampere (A) - Measures electrical current.
  - Kelvin (K) - Measures absolute temperature.
  - Candela (cd) - Measures luminous intensity.

Base Units and Derived Units

- **A unit is considered a base unit if it is one of the seven SI units: meter, second, kilogram, Ampere, mol, Kelvin, and candela.**
- *A unit is considered to be a derived unit if it is a composition of the base units.*
  - Example: Coulomb is a derived unit consisting of the base units Ampere and second; $C = A\ s$
  - Example: Newton is a derived unit consisting of the base units kilogram, meter, and second; $N = kg\ m\ s^{-2}$.

Common AS-Level Derived Units

- There are a handful of derived units you should eventually memorize:
    - Newton (Force) = kg m s^{-2}
    - Pascal (Pressure) = kg m^{-1} s^{-2}
    - Joule (Energy) = kg m^2 s^{-2}
    - Watt (Power) = kg m^2 s^{-3}
    - Hertz (Frequency) = s^{-1}
    - Coulomb (Electrical Charge) = A s
    - Volt (Voltage, Potential Difference, emf) = kg m^2 s^{-3} A^{-1}
    - Ohm (Electrical Resistance) = kg m^2 s^{-3} A^{-2}
- Do not worry about memorizing these right now!

Base vs. Derived Units and Quantities

- Kelvin - **base** *unit*, K
- Watt - **derived** *unit*, W
  - Watt may be written in **base form** as kg m^2 s^{-3}.
- A unit is in **base form** if it is written only using *base units*.
- Electric current - **base** *quantity*, I.
- Electric charge - **derived** *quantity*, Q.
  - Electric charge can be represented as I/t; however, it for simplicity it is represented as Q.
- A quantity that cannot be simplified any further is considered a **base quantity**.
  - Examples - time, current, mass, position, temperature, etc.
- A quantity that can be simplified further is considered a **derived quantity**.
  - Examples - voltage, resistance, electric charge, force, energy, momentum, etc.

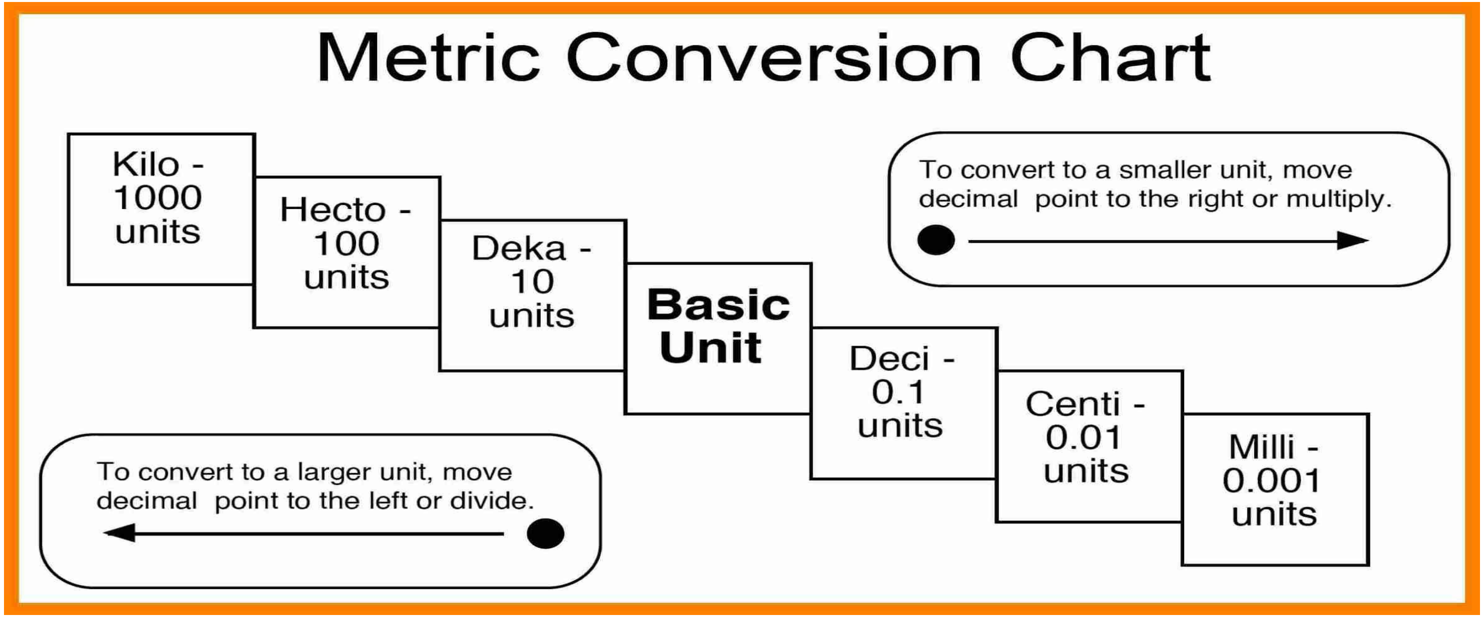
Example Problem 1

- Using the law; $W = Fx$, show the base units of the Joule (J) which is the unit of work or rather energy.
- The Joule (J) should be equal to the combined units of F and x which are force and displacement respectively.
- Hence J = units of F multiplied by units of displacement.
  - F comes from $F = ma$ hence m is measured in kg and a is measured in $m\ s^{-2}$ and x is measured in m. Combining like terms we obtain $J = kg\ m^2\ s^{-2}$.

Example Problem 2

- Using the equation; $p = mv$, show the derived units of momentum (p) is equal to Ns where N is the newton and s is the second.
- Our goal here is to show that the units of mass (m) and velocity (v) can be written as Ns (the Newton second).
  - Convert left hand side to base units: Ns = kg m s^{-1}
  - Convert right hand side to base units: kg m s^{-2} s = kg m s^{-1}
  - Notice we are missing instances of seconds on the right. If we take s^{-1} to s^{-2} we can compensate for that by adding a s to keep the integrity of the equation. *Alternatively we can simplify s^{-2} and s^{1} to make s^{-1} on the left.*
  - Hence kg m s^{-2} s = kg m s^{-2} s.

Metric System



Metric System

King Henry Died Drinking Chocolate Milk

Mnemonic	**King**	**Henry**	**Died**	**Base Unit**	**Drinking**	**Chocolate**	**Milk**
Length: Abbreviation:	Kilometer km	Hectometer hm	Decameter dam	Meter m	Decimeter dm	Centimeter cm	Millimeter mm
Weight: Abbreviation:	Kilogram kg	Hectogram hg	Decagram dag	Gram g	Decigram dg	Centigram cg	Milligram mg
Volume: Abbreviation:	Kiloliter kL	Hectoliter hL	Decaliter daL	Liter L	Deciliter dL	Centiliter cL	Milliliter mL
How many are in **1** meter/gram/liter	.001	.01	.1	1	10	100	1000
How many meters/grams/liters are in this unit?	1000	100	10	1	.1	.01	.001
	⇐ **BIGGER**				**SMALLER** ⇒		

Metric System

- Metric units can be expressed in terms of power of 10 where the base unit is 1.
  - Kilo (k) = 1×10^{3}
  - Hecto (h) = 1×10^{2}
  - Deka (da) = 1×10^{1}
  - Deci (d) = 1×10^{-1}
  - Centi (c) = 1×10^{-2}
  - Milli (m) = 1×10^{-3}

Metric System

- There are other notable units that show up that you *should* get familiar with for this course:
  - Pico (p) = 1×10^{-12}
  - Nano (n) = 1×10^{-9}
  - Micro (μ) = 1×10^{-6}
  - Mega (M) = 1×10^{6}
  - Giga (G) = 1×10^{9}
  - Tera (T) = 1×10^{12}

Converting Within Metric System

- Calculations and conversions within the metric system can be achieved by multiplying or dividing by 10 X number of times based on what position you want to move to.
- Going **down** the ladder is **multiplying**, going *up* is *dividing*.
  - Example: Convert 1 meter to centimeters.
    - Meter is 2 “spaces” from centimeter so you would multiply your quantity by 10 twice!
    - 1 m is in fact 100 cm. 1 x 10 x 10 = 100!

Converting Within Metric System

- How many km are there in 10 m?
    - Starting with 10 m, km is 3 spaces away from m so you will divide by 10, 3 times!
        - Answer: 0.01 km. This answer makes sense. If there is 0.001 km in 1 meter, then it should go without saying that 10 m will be 0.01 km.

Converting Within Metric System

- How many cm^2 are there in 1 m^2?
- We at first see that we are a high unit going to a low unit so we should use division with a multiplication of two spaces.
  - 1 x 10 x 10 = 100?
  - **Something though doesn't seem right. If you check your work using a converter we see 1 m^2 is actually 10,000 cm^2. So where did we go wrong?**

Converting Within Metric System

- How many cm^2 are there in 1 m^2?
- **When having to convert units that are square or cubic units you must honor exponent rules!**
  - $1 \times (10^1)^2 \times (10^1)^2 = 1 \times 100 \times 100 = 10{,}000$
  - Thus 1 m^2 is 10,000 cm^2 or 1.0×10^4 cm^2.
- Another way of thinking about converting between raised metric units is you perform the conversion n-extra times where n is the exponent.
- The next slide will demonstrate this idea.

Converting Within Metric System

- How many mm^3 are there in 1 m^3?
- Again, do not fall for the trap of using standard conversion and saying 1000 because that is **not correct**!
- Using the alternative method on the prior slide where we can think of the powers as additional multiplications/divisions or shuffling of the decimal points we see that:
  - N = 3 so we perform the conversion 3 times so $1m^3$ has 1.0^9 mm^3 since we move the decimal 3 times a total of 3 times!
- Makes sense?

Example Problem 3

- Convert 100 mA to μA.
  - It is worth noting to recognize where you start and where you are going. We will be going to a smaller unit hence we will be multiplying.
  - Using conversion factors we have 100 mA x (1000 μA / 1 mA).
  - Notice mA disappears and we are left with 100 x 1000 μA hence the answer is 100,000 μA.
  - If we wanted to represent this answer in A we can express it in scientific notation as 1.0 x 10^5 A.

Example Problem 4

- Convert 1000000 m^2 to km^2.
- It is worth noting to recognize where you start and where you are going. We will be going to a bigger unit hence we will be dividing.
- Using conversion factors we have 1000000 m^2 x $(1\ km / 1000\ m)^2$.
- The exponent of 2 covers us for the second conversion required as discussed prior.
- Notice m^2 and m^2 cancel and we are left with 1000000 km^2 / 1000000 and we are left with 1 km^2.

Significant Figures

- Digits in data and measurements that are used to indicate degrees of precision.
- Three major rules:
    - Non-zero digits are ALWAYS significant. [Rule 1]
    - Any zeros between two significant digits are significant. [Rule 2]
    - A final zero or final trailing zeros in decimals are significant ONLY in the decimal portion! [Rule 3]

Significant Figures

- Examples:
  - 723 has 3 significant digits due to Rule 1.
  - 13 has 2 significant digits due to Rule 1.
  - 407 has 3 significant digits due to Rule 2.
  - 4070 has 3 significant digits due to Rule 2.
    - The last 0 here is not significant because it is outside the decimal.
  - 7230.0 has 5 significant rules due to Rule 2 and Rule 3.
  - 0.007230 has 4 significant rules due to Rule 3.
  - 0.10000 has 5 significant rules due to Rule 3.

Significant Figures

- How many significant figures are in each number?
  - 3.10×10^2
  - 310
  - 0.0080100
  - 90011
  - 7230.0000

Significant Figures

- How many significant figures are in each number?
  - 3.1×10^2 [2]
  - 310 [2]
  - 0.0080100 [5]
  - 90011 [5]
  - 7230.0000 [8]

Significant Figures in Calculations

- When adding or subtracting, the number of decimal places in your result should equal the smallest number of decimal places of any term in the sum.
  - Example: 123 + 5.35 = 128
    - 123 has no decimal places so the result should have no decimal places.
  - Example: 1.002 - 0.998 = 0.004

Significant Figures in Calculations

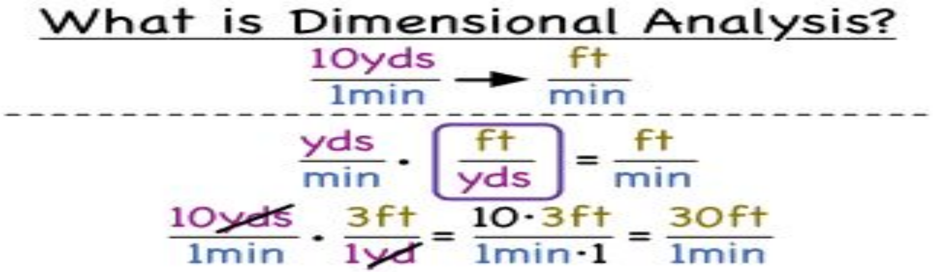
- When multiplying or dividing, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the smallest number of significant figures.
  - Example: 3.10 x 4.520 = 14.0
    - 3.10 has 3 SF and 4.520 has 4 SF.
  - Example: 507 / 1 = 500
    - 507 has 3 SF and 1 has 1 SF.

Significant Figures on the Final Exam

- Significant digits are rarely tested on the final exam; however, if they are it usually is only 1-2 marks at most.
- Usually significant digits are only tested in how you indicate your final answer.
- For example, if a question asks you to provide your answer to 3 significant digits and your calculator gives you an answer of 723.05 then the 0.05 would be thrown away and you would write down 723.

Dimensional Analysis

- Dimensional Analysis is the procedure of converting units of different measurements to other measurements or ratios of units.
    - Example: Converting miles per hour to meters per second.
- The goal is to establish fractions and ratios that allow you to cancel units until you obtain your desired goal.



Dimensional Analysis

- You can do this with different ratios too, such as shown below with mile per hour to feet per second.
- Notice that you are merely trying to cancel units out via division or multiplication.

Converting Rate Units

$$\text{Rate} = \frac{72\text{ mi}}{1\text{ hr}} \quad \frac{\cancel{\text{mi}}}{\text{hr}} \cdot \boxed{\frac{\text{ft}}{\cancel{\text{mi}}}} = \frac{\text{ft}}{\cancel{\text{hr}}} \cdot \boxed{\frac{\cancel{\text{hr}}}{\text{sec}}} = \frac{\text{ft}}{\text{sec}}$$

$$\frac{72\ \cancel{\text{mi}}}{1\ \cancel{\text{hr}}} \cdot \frac{5280\text{ ft}}{1\ \cancel{\text{mi}}} \cdot \frac{1\ \cancel{\text{hr}}}{3600\text{ sec}}$$

$$= \frac{72 \cdot 5280\text{ ft} \cdot 1}{1 \cdot 1 \cdot 3600\text{ sec}} = \frac{380{,}160\text{ ft}}{3600\text{ sec}}$$

$$= \frac{105.6\text{ ft}}{1\text{ sec}} = \boxed{105.6\text{ ft/sec}}$$

Example Problem 5

- Convert 23 radians per minute to degrees per second.
- Using conversion factors we can write an expression where our aim is to cancel units: (23 radians / 1 minute) x (180 degrees / π radians) x (1 minute / 60 seconds).
- Simplifying we get an answer of about 22 degrees per second.

Example Problem 6

- A water pump moves 1000 tonnes of water per hour. Calculate how many kg of water is pumped per second. 1 tonnes = 1000 kg.
- Using conversion factors we can write an expression where our aim is to cancel units: (1000 tonnes / 1 hour) x (1000 kg / 1 tonnes) x (1 hour / 60 minutes) x (1 minute / 60 seconds).
- Simplifying we get an answer of about 278 kg per second.

Orders of Magnitude

- The orders of magnitude are another way of categorizing measurements in powers of 10.
- The baseline for the order of magnitude is 10^0 which is equal to 1.
- Therefore 1 m has an order of magnitude of 10^0.

Orders of Magnitude

- The size of a kilometer long dirt bike road is 1 times 10^3 since 1 km is 1000 m. Makes sense?
- The thickness of a dime is about 1 mm so it has an order of magnitude of 1 times 10^{-3} since 1 mm is 0.001 m!
- Do we happen to see a pattern?

Table 8.5 Items to Illustrate When Making Powers of Ten Posters

Distance	Comparison (Approximate)
10^{0} m	Distance from floor to door knob
10^{-1} m	Width of hand
10^{-2} m	Width of fingernail on smallest finger
10^{-3} m	Thickness of a U.S. dime
10^{-4} m	Length of a dust mite
10^{-5} m	Diameter of human red blood cells
10^{-6} m	Diameter of small bacteria
10^{-7} m	Length of a virus
10^{-8} m	Thickness of bacteria flagellum
10^{-9} m	Width of DNA helix
10^{-10} m	Width of ice or quartz cell
10^{-11} m	Radius of a hydrogen atom
10^{-12} m	Wavelength of X-rays
10^{-13} m	Wavelength of an electron
10^{-14} m	Diameter of a nucleus
10^{-15} m	Diameter of a proton
10^{-16} m	One-tenth the diameter of a proton
10^{-17} m	One-hundredth the diameter of a proton
10^{-18} m	Radius of an electron

Increasing Size ↑

Distance	Comparison (Approximate)
10^{26} m	Radius of observable universe
10^{25} m	Distance to the 3C273, brightest quasar
10^{24} m	Distance to the nearest large supercluster
10^{23} m	Distance to galaxies beyond our local group
10^{22} m	Distance to Andromeda galaxy
10^{21} m	Diameter of the disc of the Milky Way
10^{20} m	Diameter of the Small Magellanic Cloud
10^{19} m	Approximate thickness of the Milky Way
10^{18} m	Diameter of a typical globular cluster
10^{17} m	Distance from Earth to Vega
10^{16} m	Inner radius of Oort cloud
10^{15} m	100 × diameter of the solar system
10^{14} m	10 × diameter of the solar system
10^{13} m	Diameter of solar system
10^{12} m	Distance from Sun to Saturn
10^{11} m	Distance from Sun to Venus
10^{10} m	One half the distance light travels in a minute
10^{9} m	Diameter of the Sun
10^{8} m	Diameter of Saturn
10^{7} m	North Pole to equator
10^{6} m	Length of California (north to south)
10^{5} m	Length of Connecticut (north to south)
10^{4} m	Depth of Mariana Trench, deepest point
10^{3} m	One kilometer; 2.5 times around a track
10^{2} m	One side of a running track
10^{1} m	Distance for a first down in football
10^{0} m	Distance from floor to door knob

Orders of Magnitude

- Examples:
  - The width of a DNA helix is a nanometer.
    - Since nanometer is -9 power, it has an order of magnitude of -9 or 1 times 10 to the -9 power.
  - The distance from the Sun to Saturn is a terameter.
    - Since terameter is 12 power, it has an order of magnitude of 12 or 1 times 10 to the 12 power.

Orders of Magnitude

- The order of magnitude for an estimate, calculation, or value is simply the exponent in the 1 times 10 to the n-power.
- Once again, the base unit is order of magnitude 0 because 1 times 10 to the 0th power is simply 1.